arXiv:0711.4784v2 [cond-mat.stat-mech] 6 May 2008

Transport reversal in a delayed feedback ratchet

M. Feito*
Departamento de Física Atómica, Molecular y Nuclear, Universidad Complutense de Madrid, Avenida Complutense s/n, 28040 Madrid, Spain.

F. J. Cao†
Departamento de Física Atómica, Molecular y Nuclear, Universidad Complutense de Madrid, Avenida Complutense s/n, 28040 Madrid, Spain and LERMA, Observatoire de Paris, Laboratoire Associé au CNRS UMR 811 2, 61, Avenue de l'Observatoire, 75014 Paris, France.

Feedback flashing ratchets are thermal rectifiers that use information on the state of the system to operate the switching on and off of a periodic potential. They can induce directed transport even with symmetric potentials thanks to the asymmetry of the feedback protocol. We investigate here the dynamics of a feedback flashing ratchet when the asymmetry of the ratchet potential and of the feedback protocol favor transport in opposite directions. The introduction of a time delay in the control strategy allows one to nontrivially tune the relative relevance of the competing asymmetries leading to an interesting dynamics. We show that the competition between the asymmetries leads to a current reversal for large delays. For small ensembles of particles current reversal appears as the consequence of the emergence of an open-loop like dynamical regime, while for large ensembles of particles it can be understood as a consequence of the stabilization of quasiperiodic solutions. We also comment on the experimental feasibility of these feedback ratchets and their potential applications.

PACS numbers: 05.40.-a, 02.30.Yy

I. INTRODUCTION

Brownian motors or ratchets are spatially periodic systems that are able to induce direct transport rectifying thermal fluctuations. Two conditions are generally sufficient for the emergence of direct transport in these systems: breaking of thermal equilibrium and breaking of spatial inversion symmetry [1]. These systems permit one to get an insight into non-equilibrium processes and are receiving increasing interest also due to their applications in nanotechnology and biology [1, 2, 3].

Flashing ratchets are devices that rectify the motion of Brownian particles by subjecting them to a spatially periodic potential that is alternatively switched on and off. Open-loop flashing ratchets operate without regard to the state of the system (open-loop control) implementing a periodic or random switching to rectify thermal fluctuations by taking advantage of the asymmetry of the potential [4, 5, 6]. On the contrary, feedback ratchets (or closed-loop ratchets) use information on the particle distribution of the system to operate [7, 8, 9, 10, 11, 12, 13], and the asymmetry of the feedback control protocol is able to induce a directed transport even for symmetric ratchet potentials. For instance, in the so-called maximization of the center-of-mass velocity protocol [7] the controller switches on the potential only if switching on would imply a positive displacement for the center-of-mass position (i.e., if the net force with the potential on would be positive). Feedback flashing ratchets have been recently suggested as a mechanism to explain the stepping motion of the two-headed kinesin [14]. In another context, a feedback scheme has been used to perform control of chaotic trajectories in inertia ratchets [15].

Feedback flashing ratchets could be experimentally implemented monitoring the positions of a set of Brownian particles [16, 17, 18] and subsequently using the information gathered to decide whether to switch on or off a ratchet potential according to a giving protocol. This experimental design will have to deal with a finite time lag between the collection of the information about the state of the system and the action because of the time interval needed for the measurement, transmission and processing of the information [19, 20]. Time delays in the feedback also appear naturally in complex systems with self regulating mechanisms (see [21, 22] and references therein). It is also remarkable for the ability of controlling chaos and improving coherence in excitable systems under delayed feedback [23, 24]. The

*Electronic address: feito@fis.ucm.es
†Electronic address: francao@fis.ucm.es

feasibility of nanotechnological feedback flashing ratchet devices and their performance under the presence of a time delay has been analyzed very recently in Refs. [12, 13]. In those works, and also in previous ones [7, 8, 9, 10, 11], the two sources of spatial asymmetry involved, namely the feedback control and the shape of the potential, *cooperate* with the aim of maximizing the performance of the system. On the contrary, in this paper we investigate the effects of the *competition* between the potential asymmetry and the control asymmetry in a delayed feedback ratchet.

We have observed a rich dynamics that includes transport reversal. The inversion of the current direction upon the variation of the system parameters is a well-known phenomenon in Brownian motors that can be produced by varying the characteristics of the non-equilibrium fluctuations [25, 26] or the parameters of the time-dependent perturbation that drives the system out of equilibrium [27, 28, 29, 30, 31]. It also appears in other ratchet-like systems, such as deterministic inertial ratchets [32, 33]. The phenomenon of current reversal has great importance in particle separation devices [34], and in biology systems [35]. In our present study current reversal is achieved just by varying the time delay of the system.

We start below with the description of the collective flashing ratchet and the delayed feedback protocol that we consider. In the next section, Sec. III, the evolution equations of the system are solved by Langevin dynamics simulations and the rich dynamics encountered (transport reversal, quasi-periodic modes of oscillation, multistability) is analyzed. We finally review and further discuss in Sec. IV the implications of the results.

II. MODEL

The feedback ratchet that we consider consists of N Brownian particles at temperature T in a periodic potential $V(x)$. The force acting on the particles is $F(x) = -V'(x)$, where the prime denotes the spatial derivative. The state of this system is described by the positions $x_i(t)$ of the particles satisfying the overdamped Langevin equations

$$\gamma \dot{x}_i(t) = \alpha(t) F(x_i(t)) + \xi_i(t); \quad i = 1, \dots, N, \tag{1}$$

where γ is the friction coefficient (related to the diffusion coefficient D through Einstein's relation $D = k_B T/\gamma$), $\xi_i(t)$ are Gaussian white noises of zero mean and variance $\langle \xi_i(t) \xi_j(t') \rangle = 2\gamma k_B T \delta_{ij} \delta(t - t')$, and $\alpha(t)$ stands for the action of the controller. The feedback policy uses the sign of the net force per particle,

$$f(t) = \frac{1}{N} \sum_{i=1}^{N} F(x_i(t)), \tag{2}$$

as follows: The controller measures the sign of the net force and, after a time τ, switches the potential on ($\alpha = 1$) if the net force was positive or switches the potential off ($\alpha = 0$) if the net force was negative. Therefore, the delayed control protocol considered is

$$\alpha(t) = \begin{cases} \Theta(f(t-\tau)) & \text{if } t \geq \tau, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

with Θ the Heaviside function [$\Theta(x) = 1$ if $x > 0$, else $\Theta(x) = 0$]. We have used a sawtooth potential of period L, i.e. $V(x) = V(x + L)$, height V_0, and asymmetry parameter a:

$$V(x) = \begin{cases} \frac{V_0}{a} \frac{x}{L} & \text{if } 0 \leq \frac{x}{L} \leq a, \\ V_0 - \frac{V_0}{1-a} \left(\frac{x}{L} - a \right) & \text{if } a < \frac{x}{L} \leq 1. \end{cases} \tag{4}$$

The height V_0 of the potential is the difference between the value of the potential at the minimum and at the maximum, while aL is the distance between the minimum and the maximum consecutive positions (Fig. 1). Thus when $a < 1/2$ both the asymmetry of the potential and the feedback protocol favor transport in the same direction, whereas when $a > 1/2$ there is a competition between them. We consider here the latter case.

III. RESULTS

We have performed numerical simulations of the Langevin equations (1) by using an Euler-Maruyama scheme [36], which reveals different dynamics of the collective ratchet for different ensemble sizes when the delayed feedback is present. We distinguish between few particles (including $N = 1$ as a particular case) and many particles. Following previous works [7], we refer to the few particle case when the average long-time limit velocity of the center-of-mass,

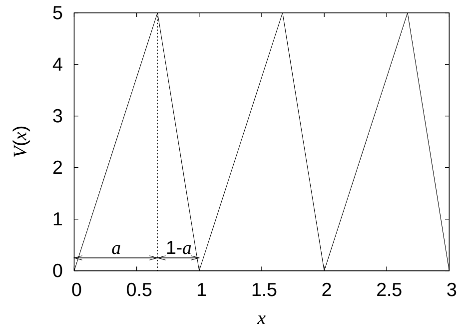

FIG. 1: Sawtooth potential [Eq. (4)] of height $V_0 = 5k_BT$ and asymmetry parameter $a = 2/3$. Units: $L = 1$ and $k_BT = 1$.

$\langle \dot{x}_{\rm cm} \rangle$, is greater for the non-delayed feedback maximization protocol than for the optimal open-loop protocol, and many particle case otherwise. Typically the frontier between these regimes corresponds to $N = 10^2 - 10^3$ particles for potential heights of the order of $5k_BT$ or greater.

Let us begin our analysis with the non-delayed ($\tau = 0$) feedback ratchet.

A. Non-delayed feedback ratchet

For *one particle* ($N = 1$) the exact expression for the average velocity can be obtained by solving a Fokker-Planck equation with the proper effective potential that includes the action of the controller. This expression has been derived in [7], and is indeed valid for any asymmetry parameter $0 < a < 1$. It gives a positive flux that grows as $DV_0/(k_BTL)$ for small potential heights ($V_0 \lesssim k_BT$), and tends to the finite value $2D/(a^2L)$ for large potential heights ($V_0 \gg k_BT$). The fact that the flux goes to a constant value for large potential heights is a direct consequence of the overdamped nature of the ratchet. We remark that enlarging the value of the ratio $V_0/(k_BT)$ corresponds to effectively diminish the intensity of the white noise that accounts for thermal fluctuations.

For the collective ratchet compounded of a *few particles* an approximation for the center-of-mass velocity can be obtained assuming a purely stochastic behavior (see Ref. [7] for details). As the magnitude of the fluctuations of the force are of the order of the inverse of the square-root of the number of particles the stochastic approximation predicts the $\langle \dot{x}_{\rm cm} \rangle \sim 1/\sqrt{N}$ decay observed in our simulations for any asymmetries. In fact this qualitative behavior remains valid for any number of particles (including *many particles*) provided the asymmetry is $a > 1/2$. In this latter case the potential asymmetry acts against the feedback protocol, which tries to favor positive currents, and then the potential is turned on in very small intervals of time as the controller rapidly switches it off. See Fig. 2. Thus the systems dynamics is effectively stochastic, contrary to the many particle case with cooperating asymmetries, where switches are slower and allow the system to have enough time to evolve in a quasideterministic way [7]. Therefore when the potential asymmetry competes against the feedback the flux decays with the number of particles as $1/\sqrt{N}$ even for many particles, contrary to the much slower $1/\ln N$ dependence observed when both asymmetries cooperate [7].

In any case the non-delayed protocol always gives a positive flux because it only switches on when it implies a positive displacement of the center-of-mass position.

B. Delayed feedback ratchet

The presence of a lag time in the control can cause negative currents and complicated dynamics that depends on the number of particles. Let us first study the few particle case (including one particle).

1. *Few particles*

When the control protocol presents a time delay the system performs worse because the delayed action of the controller implies some wrong actions. Moreover, for large time delays the controller is unable to surmount the potential shape asymmetry and eventually the net current becomes negative. See Figs. 3 and 4.

For increasing time delays the correlation between the present sign of the net force and the measured sign that the controller actually uses decreases. Thus the controller action begins to be uncorrelated to the present state of the

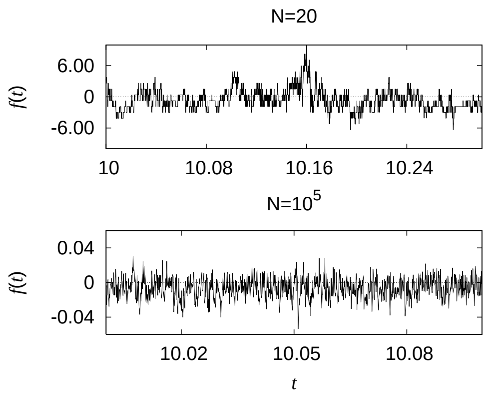

FIG. 2: Evolution of the net force per particle for the non delayed feedback ratchet for $N = 20$ (few particles) and $N = 10^5$ (many particles). The potential is "on" only in the time intervals such that $f(t)$ is positive. Parameters of the potential: $V_0 = 5k_BT$ and $a = 2/3$. Units: $L = 1$, $D = 1$, and $k_BT = 1$.

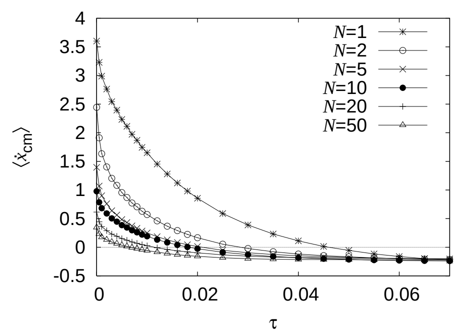

FIG. 3: Center-of-mass velocity $\langle \dot{x}_{\rm cm} \rangle$ as a function of the time delay τ for different numbers N of particles under the few particle regime. Parameters of the potential: $V_0 = 10k_BT$ and $a = 2/3$. Units: $L = 1$, $D = 1$, and $k_BT = 1$.

system and it effectively begins to act as an open-loop ratchet [12]. In fact, for large delays the correlation between the state of the system and the measured retarded state is negligible and the negative flux becomes independent of the delay, because the ratchet is effectively open-loop controlled; see Fig. 3. Therefore transport reversal appears here as a consequence of the competition between the asymmetry of the ratchet potential and the inherent asymmetry of the protocol. We stress that the influence of the asymmetry of the feedback protocol itself is not tuned here trivially, but changing the delay τ in the control. Other ways of tuning the influence of the feedback protocol could not lead to current reversal. For example, it can be shown that a feedback protocol that switches on/off following the maximization protocol but with a probability of error $0 < p < 1/2$ does not enable negative fluxes even for

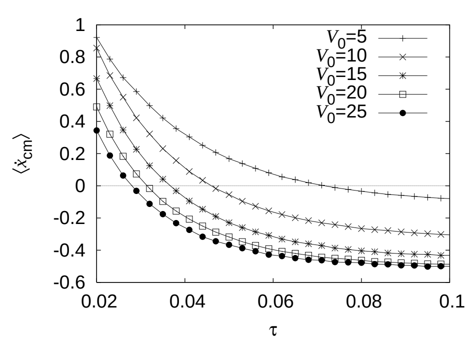

FIG. 4: One particle velocity versus the time delay for different heights V_0 of the potential and asymmetry parameter $a = 2/3$. Units: $L = 1$, $D = 1$, and $k_BT = 1$.

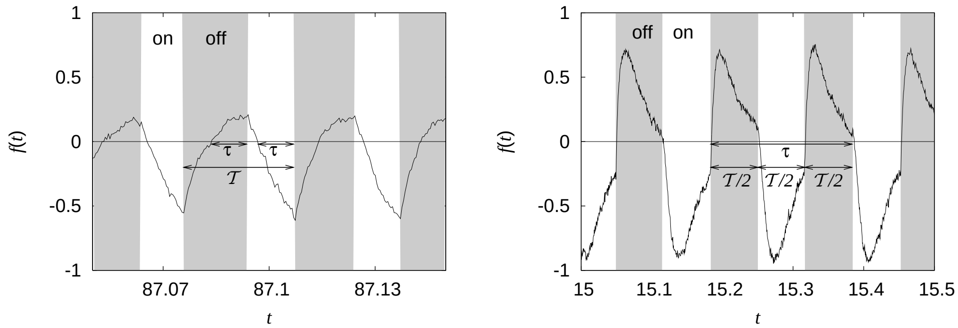

FIG. 5: Evolution of the net force per particle for time delays $\tau = 0.01$ (left) and $\tau = 0.20$ (right) in the many particle case ($N = 10^5$). White background regions stand for "on" potential and gray background regions for "off" potential. Parameters of the potential: $V_0 = 5k_BT$ and $a = 2/3$. Units: $L = 1$, $D = 1$, and $k_BT = 1$.

asymmetries $a > 1/2$ (see Ref. [9]).

For the delayed protocol considered the critical value of the delay that gives zero current (thus the current is positive for smaller delays and negative for larger delays) is related with the characteristic time in which the information about the state of the system is effectively lost, so the delayed maximization protocol is not able to achieve its goal of producing a positive current for delays larger than the critical one. Increasing the height V_0 of the potential implies a faster dynamics. Therefore the critical delay is expected to decrease with the height of the potential, in agreement with our simulations; see Fig. 4. It is important to note that this critical delay tends to a constant *nonzero* value as $V_0 \to \infty$. The reason is the same that makes the absolute value of the flux in both closed-loop and open-loop ratchets does not grow indefinitely as the potential goes up. These fluxes tend to a finite value because the time spent by the particles in diffusing during the off potential state goes to a constant in the absence of inertia. Note also that the critical delay decreases with the number of particles (see Fig. 3.)

Let us now study the many particle case, which exhibits a completely different dynamics.

2. Many particles

For large ensembles of particles ($N > 10^2 - 10^3$) the flux is nearly zero in the non-delayed protocol (see Sec. III A), but the introduction of a time delay stabilizes quasiperiodic solutions that give noticeable negative currents. We have found that, after a transient time, the delayed control allows the system to synchronize into a stable mode of oscillation such that the net force per particle evolves quasi-periodically. The evolution is not strictly periodic due to the stochastic nature of the dynamics. See Fig. 5.

For small delays the on and off times of the non-delayed dynamics (Sec. III A) are enlarged owing to the delay, and the net force per particle evolves with a more regular pattern (Fig. 5, left). When the potential is switched on the net force per particle begins to diminish (because the potential asymmetry acts against the feedback protocol) and rapidly gets negative, but the potential still remains 'on' during a time τ after the force changed its sign. On the other hand, when the potential is switched off the net force grows, becomes positive, and induces an 'on' switching a time τ later. The result is a quasiperiodic dynamics with an small quasiperiod $\mathcal{T} > 2\tau$; see Fig. 5 (left). We highlight that these types of solution are only observed for asymmetries $a > 1/2$, as they are consequences of the competing asymmetry of the potential; they do not appear for asymmetries $a < 1/2$ that support positive transport and exhibit a different behavior related with the enlargement of the tails of the net force per particle [12, 13].

For larger delays there are stable solutions of quasiperiods $\mathcal{T} = 2\tau/(2n+1),\ n = 0, 1, \ldots,$ i.e., solutions that contain an odd number of semiperiods $\mathcal{T}/2$ in the time delay τ. See Fig. 5 (right) for instance. The competing asymmetry of the potential causes the stabilization of those solutions where, due to the delay, the controller switches on when the present net force is negative and switches off when it would be positive, that is, the controller acts contrary to its intentions and gives a negative flux. These branches are the counterparts of the solutions of quasiperiods $\mathcal{T} = \tau/n,\ n = 1, 2, \ldots$ (τ containing an even number of semi-quasiperiods) observed for asymmetries $a < 1/2$ [12]. The difference of one semiperiod is due to the effectively reversed operation of the controller caused by the combined effect of the competing asymmetry of the potential and the delay. Some of these branches are plotted in Fig 6 for both the cases of cooperation (positive currents) and competition (negative currents) of asymmetries.

It is important to note that the average velocity for all these branches can be reexpressed in terms of one of them. Let us define $g(\tau) := \langle \dot{x}_{\mathrm{cm}} \rangle_\tau(\tau)$ as the average velocity for the branch of period $\mathcal{T} = \tau$, which is present for

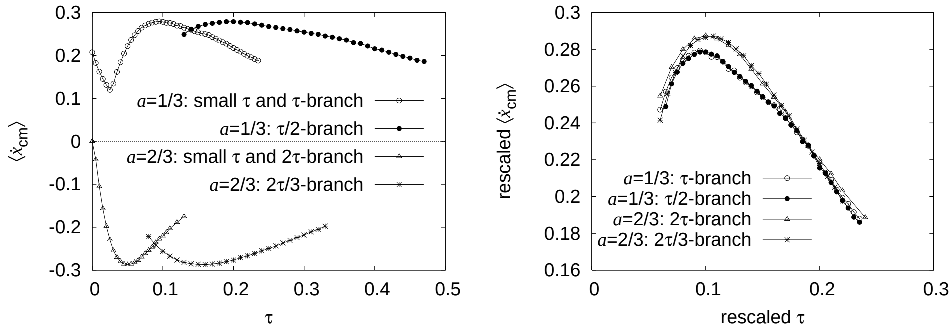

FIG. 6: Left panel: Center-of-mass velocity $\langle \dot{x}_{\rm cm} \rangle$ versus the delay τ in the many particle case ($N = 10^5$). The region of small delays and the first two branches for asymmetries parameters $a = 1/3$ (positive flux) and $a = 2/3$ (negative flux) are plotted for height of the potential $V_0 = 5k_BT$. Units: $L = 1$ and $k_BT = 1$. Right panel: First two branches for asymmetries $a = 1/3$ and $a = 2/3$ for potential height $V_0 = 5k_BT$ and $N = 10^5$ particles, rescaled according to scaling laws (5) and (6).

cooperative potential asymmetry. For these asymmetries, $a < 1/2$, we showed in Ref. [12] that the average velocities of the branches of periods $\mathcal{T} = \tau/n$ are given by

$$\langle \dot{x}_{\rm cm} \rangle_{\frac{\tau}{n}}(\tau) = g\left(\frac{\tau}{n}\right) \qquad \text{for } a < 1/2. \tag{5}$$

On the contrary, for competing asymmetries, $a > 1/2$, we have found here that the solutions have quasiperiods $\mathcal{T} = 2\tau/(2n+1)$, and furthermore, the average velocities of these branches are given by

$$\langle \dot{x}_{\rm cm} \rangle_{\frac{2\tau}{2n+1}}(\tau) = -g\left(\frac{2\tau}{2n+1}\right) \qquad \text{for } a > 1/2. \tag{6}$$

Consequently, given one of the branches all the others can be predicted; see Fig. 6. This also implies that the analytical results obtained in Ref. [12] for cooperative potential asymmetry ($a < 1/2$) are directly extended to the competing potential asymmetry case ($a > 1/2$), just using the relation in Eq. (6).

IV. CONCLUSIONS

We have studied the performance of feedback flashing ratchets when there is competition between the asymmetry in the potential and the asymmetry in the control protocol, and we have also studied the effects of tuning their relative influences in the dynamics by the introduction of a time delay. An experimental realization of a flashing ratchet has been performed in [16] by using polystyrene latex spheres of diameters $d \simeq 0.25 - 1\ \mu m$ in an aqueous solution [viscosity $\eta \simeq 10^{-3}\ Pa \cdot s$; $D = k_BT/(3\pi\eta d)$] exposed to a sawtooth dielectric potential of period $L \simeq 50\ \mu m$. This experimental setup can be modified to become an experimental realization of a feedback flashing ratchet by monitoring the particles with a conventional charge-coupled device (CCD) of about 30 fps and processing the images to switch on or off the ratchet potential in accordance with the particle positions. The time delays considered here are introduced by delaying the action of the controller a time between $\tau = 0.01L^2/D \sim 10s$ and $\tau = 0.5L^2/D \sim 500s$. (For a more detailed discussion see Ref. [12].) Indeed a sophisticated feedback control has been recently implemented in Ref. [18], where images of a Brownian particle are acquired on a high-sensitivity CCD of up to 300 fps and thereafter a software processes the information to extract the position of the particle and apply a feedback voltage. On the other hand, we highlight that the viscous friction coefficient γ depends on the shape and the size of the Brownian particle. Thus, as the adimensional delay must be multiplied by the factor $L^2/D = \gamma L^2/k_BT$ in order to recover physical units, Brownian particles of different shape and size respond differently to a given time delay. This effect could be useful for separating different kinds of macromolecules.

We have seen that the performance of the system with competing asymmetries differs significatively from its counterpart ratchet with cooperating asymmetries. In the absence of delay the competition of asymmetries implies a decay of the current with the size of the ensemble much stronger than in the cooperative case ($1/\sqrt{N}$ vs $1/\ln N$). In the presence of delay the dynamics becomes richer with a current reversal for large delays. In the few particle regime the change from positive to negative current can be understood as a change from a purely closed-loop control to an effective open-loop control. On the other hand, in the many particle case the negative current regime appears for large enough delays as the consequence of the stabilization of several branches of quasiperiodic solutions. These stable branches have the opposite sign and are one semiperiod displaced with respect to those obtained for cooperating

asymmetries, they also have a direct relation with them that allows the extension for the competing asymmetries case of the analytical results found in Ref. [12] for cooperative asymmetries.

Acknowledgments

We acknowledge financial support from the Ministerio de Ciencia y Tecnología (Spain) through the Research Project FIS2006-05895. In addition, M.F. thanks the Universidad Complutense de Madrid (Spain) and F.J.C. thanks ESF Programme STOCHDYN for their financial support.

---

[1] P. Reimann, Phys. Rep. **361**, 57 (2002).
[2] H. Linke, Appl. Phys. A **75**, 167 (2002).
[3] E. R. Kay, D. Leigh, and F. Zerbetto, Angew. Chem. Int. Ed. **46**, 72 (2007).
[4] A. L. R. Bug and B. J. Berne, Phys. Rev. Lett. **59**, 948 (1987)
[5] A. Ajdari and J. Prost, C. R. Acad. Sci. Paris II **315**, 1635 (1992).
[6] R. D. Astumian and M. Bier, Phys. Rev. Lett. **72**, 1766 (1994).
[7] F. J. Cao, L. Dinis and J. M. R. Parrondo, Phys. Rev. Lett. **93**, 040603 (2004).
[8] M. Feito and F. J. Cao, Eur. Phys. J. B **59**, 63 (2007).
[9] F. J. Cao, M. Feito, and H. Touchette, *Information and flux in a feedback controlled Brownian ratchet*, arXiv:cond-mat/0703492 (2007).
[10] L. Dinis, J. M. R. Parrondo, and F. J. Cao, Europhys. Lett. **71**, 536 (2005).
[11] M. Feito and F. J. Cao, Phys. Rev. E **74**, 041109 (2006).
[12] M. Feito and F. J. Cao, Phys. Rev. E. **76**, 061113 (2007).
[13] E. M. Craig, B. R. Long, J. M. R. Parrondo, and H. Linke, Europhys. Lett. **81**, 10002 (2008).
[14] M. Bier, Biosystems **88**, 301 (2007).
[15] U. E. Vincent, A. N. Njah, and J. A. Laoye, Physica D **231**, 130 (2007).
[16] J. Rousselet, L. Salome, A. Ajdari, and J. Prost, Nature **370**, 446 (1994).
[17] C. Marquet, A. Buguin, L. Talini, and P. Silberzan, Phys. Rev. Lett. **88**, 168301 (2002).
[18] A. E. Cohen and W. E. Moerner, Proc. Natl. Acad. Sci. **103**, 4362 (2006).
[19] R. F. Stengel, *Optimal Control and Estimation* (Dover, New York, 1994).
[20] J. Bechhoefer, Rev. Mod. Phys. **77**, 783 (2005).
[21] G. A. Bochanov and F. A. Rihan, J. Comput. Appl. Math. **125**, 183 (2000).
[22] T. D. Frank, Phys. Rev. E **71**, 031106 (2005).
[23] E. Schöll and H. G. Schuster (ed), *Handbook of Chaos Control (second completely revised and enlarged edition)*, Wiley (2007).
[24] T. Prager, H-P Lerch, L. Schimansky-Geier, and E. Schöll, J. Phys. A **40**, 11045 (2007).
[25] C. R. Doering, W. Horsthemke, and J. Riordan. Phys. Rev. Lett. **72**, 2984 (1994).
[26] M. M. Millonas and M. I. Dykman, Phys. Lett. A **185**, 65 (1994).
[27] R. Bartussek, P. Hänggi and J. G. Kissner, Europhys. Lett. **28**, 459 (1994).
[28] D. Dan, M.C. Mahato, and A. Jayannavar, Phys. Rev. E **63**, 056307 (2001).
[29] B. Ai, L. Wang, and L. Liu, Phys. Rev. E **72**, 031101 (2005).
[30] M. Bier and R. D. Astumian, Phys. Rev. Lett. **76**, 4277 (1996).
[31] J. F. Chauwin, A. Ajdari, and J. Prost, Europhys. Lett. **32**, 373 (1995).
[32] J. L. Mateos, Phys. Rev. Lett. **84**, 258 (2000).
[33] M. Barbi and M. Salerno, Phys. Rev. E **62**, 1988 (2000).
[34] C. Kettner, P. Reimann, P. Hänggi and F. Müller, Phys. Rev. E **61** 312 (2000).
[35] U. Henningsen and M. Schliwa, Nature **389**, 93 (1997).
[36] P. E. Kloeden and E. Platen, *Numerical Solution of Stochastic Differential Equations*, Springer (1992).